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10MCA12

First Semester MCA Degree Examination, February 2013

Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Determine the sets A and B, given that $A - B = \{1, 2, 4\}$; $B - A = \{7, 8\}$ and $A \cup B = \{1, 2, 4, 5, 7, 8, 9\}$ **(04 Marks)**

b. For any two sets A and B, prove that

i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ **(04 Marks)**

c. A survey of 500 television viewers of a sports channel produced the following information: 285 watch cricket, 195 watch hockey, 115 watch football, 45 watch cricket and football, 70 watch cricket and hockey, 50 watch hockey and football and 50 do not watch any of the three kinds of games. i) How many viewers in the survey watch all three kinds of games? ii) How many viewers watch exactly one of the sports? **(06 Marks)**

d. A computer service company has 300 programmers. It is known that 180 of these can program in Pascal, 120 in FORTRAN, 30 in C++, 12 in Pascal and C++, 18 in FORTRAN and C++, 12 in Pascal and FORTRAN, and 6 in all three languages.

i) If a programmer is selected at random, what is the probability that she can program in exactly two languages?

ii) If two programmers are selected at random, what is the probability that they can (1) both program in Pascal? (2) Both can program only in Pascal? **(06 Marks)**

2 a. Find the possible truth values of p, q and r in the following cases:

i) $p \to (q \vee r)$ is false ii) $p \wedge (q \to r)$ is true. **(05 Marks)**

b. Define tautology. Prove that, for any propositions p, q, r the compound proposition.

$\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ is a tautology. **(05 Marks)**

c. Prove the following logical equivalences without using truth tables:

i) $p \vee \{p \wedge (p \vee q)\} \Leftrightarrow p$ ii) $[(\neg p \vee \neg q) \to (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$ **(05 Marks)**

d. Test the validity of the following arguments:

If I study,. I will not fail in the examination.
If I do not watch TV in the evenings, I will study.
I failed in the examination.
$\therefore$ I must have watched TV in the evenings. **(05 Marks)**

3 a. Suppose the universe consists of all integers. Consider the following open statements:

$p(x) : x \leq 3$, $q(x) : x + 1$ is odd , $r(x) : x > 0$

Write down the truth values of the following:

i) $p(2)$ ii) $\neg q(4)$ iii) $p(-1) \wedge q(1)$ iv) $\neg p(3) \vee r(0)$ v) $p(0) \to q(0)$ vi) $p(1) \leftrightarrow \neg q(2)$ **(06 Marks)**

b. Negate and simplify the following:

i) $\exists x_1 \{p(x) \vee q(x)\}$ ii) $\forall x_1 \{p(x) \to q(x)\}$ iii) $\exists x_1 [\{p(x) \vee q(x)\} \to r(x)]$ **(07 Marks)**

c. Give (i) 1 direct proof, (ii) an indirect proof, and (iii) proof by contradiction, for the following statement:

"If n is an odd integer, then n +11 is an even integer". **(07 Marks)**

4 a. Prove by mathematical induction, that

$1^2 + 3^2 + 5^2 + \ldots\ldots\ldots + (2n-1)^2 = \frac{1}{3} n(2n-1)(2n+1)$ **(05 Marks)**

b. Find an explicit definition of the sequence defined recursively by
$a_n = 2a_{n-1} + 1$ with $a_1 = 7$ for $n \geq 2$ (05 Marks)

c. The Fibonacci numbers ate defined recursively by $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Evaluate F_2 to F_6. (05 Marks)

d. State the pigeonhole principle. Show that if any 5 numbers from 1 to 8 are chosen, then two of them have their sum equal to 9. (05 Marks)

5 a. Define Cartesian product of two sets. For any tree non-empty sets A, B, C prove that
$A \times (B - C) = (A \times B) - (A \times C)$ (05 Marks)

b. Let A = {1, 2, 3} and B = {2, 4, 5}. Determine
i) the number of relations from A to B.
ii) the number of relations from A to B that contain (1, 2) and (1, 5). (05 Marks)

c. Let A = {1, 2, 3, 4, 6} and R be relation on A defined by aRb if and only of 'a is a multiple of b'. Represent the relation R as a matrix and draw its digraph. (05 Marks)

d. Let A = {1, 2, 3, 4, 5}. Define a relation R on A×A by $(x_1y_1)R(x_2y_2)$ if and only if $x_1+y_1 = x_2+y_2$. i) Verify that R is an equivalence relation on A × A. ii) Determine the equivalence class [(1, 3)]. (05 Marks)

6 a. If R is a relation on the set A = {1, 2, 3, 4} defined by xRy is 'x divides y'. Prove that (A, R) is a poset. Draw its Hasse diagram. (05 Marks)

b. Let A = {x / x is real and x ≥ - 1}, and B = {x / x is real and x ≥ a}. Consider f : A→B defined by $f(a) = \sqrt{a+1}$, for a ∈ A. Show that f is invertible and determine f^{-1}. (05 Marks)

c. Consider the functions f and g defined by f(x) = g and $g(x) = x^2 + 1$ for all x ∈ R. Find gof, fog, f^2 and g^2. (05 Marks)

d. Let A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5, 6}.
i) Find how many functions are there from A to B. How many of these are one-one? How many are onto?
ii) Find the number of onto functions from B to A. (05 Marks)

7 a. Define abelian group. Prove that a group G is abelian if and only if $(ab)^2 = a^2b^2$ for all a.b∈G. (05 Marks)

b. State and prove Lagrange's theorem. (05 Marks)

c. Define group homomorphism. Let f be a homomorphism from a group G_1 to a group G_2. Prove that $f(a^{-1}) = [f(a)]^{-1}$ for all $a \in G_1$. (05 Marks)

d. A binary symmetric channel has probability p = 0.05 of incorrect transmission. If the word C = 011011101 is transmitted, what is the probability that i) single error occurs, ii) a double error occurs. (05 Marks)

8 a. Define group code. Consider the encoding function $E: Z_2^2 \rightarrow Z_2^6$ of the triple repetition code, given by E(00) = 000000, E(10) = 101010, E(01) = 010101, E(11) = 111111. Prove that C = {000000, 101010, 010101, 111111} is a group code. (06 Marks)

b. The generator matrix for an encoding function $E: Z_2^3 \rightarrow Z_2^6$ is given by $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$

i) Find the code words assigned to 110 and 010. ii) Obtain the associated parity-check matrix and hence decode the received word: 110110. (07 Marks)

c. Prove that the set Z with binary operations ⊕ and ⊙ defined by

$x \oplus y = x + y - 1$, $x \odot y = x + y - xy$ is a ring. (07 Marks)

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